Conformable Fractional Triple Laplace Transform and Its Applications to Mboctara Equations

Amjad E. Hamza*

Department of Mathematic, College of Sciences, University of Ha'il, Ha'il 2440, Saudi Arabia

* *Corresponding author; E-mail: ae.hamza@uoh.edu.sa*

Abstract:
This article initiates the study of a new conformable triple Laplace transform. Single and double conformable Laplace transforms are utilized. Some fundamental properties of the newly proposed transform are established. Some significant results are also proved by using the obtained properties. Additionally, the new transform is applied to solve Mboctara equations. By using the Mathematica the exact and proposed method solution of all the example at various fractional orders are shown in the tables and the obtained solution is compared graphically with its exact solution.

Key words: Laplace transform; conformable triple Laplace transform; Mboctara equation.

1. INTRODUCTION

Partial differential equations (PDEs) have long been regarded as one of the most complicated issues in mathematics. A novel type of non-homogeneous partial differential equation known as the Mboctara equation which is useful for analyzing the characteristics of the collective motion of microparticles in a material. Particularly in material science, one can use the Mboctara equation to analyze any material by examining its structural and geometrical framework (Srikumar Panda et al. 2020).

The classification of PDEs with respect to certain constraints is a widely studied topic. One of the reasons of doing research in this direction is to regulate the quantity and category of the constraints needed to ascertain whether the problem is posed effectively and possess a model solution. The solution procedure is entirely predicated on the classification of PDEs. It is considered that the Laplace transform is extensively applied to both linear and nonlinear equations (AE Hamza, AKH Sedeeg et al. 2023, AE Hamza, R Khalil et al. 2023, AKH Sedeeg 2023, Ozan Ozkan, Ali Kurt 2020, Ram Shiromani 2018, Ozan Özkan , Ali Kurt 2018, Abdon Atangana 2013, Ranjit R. Dhunde et al. 2013, Hassan Eltayeb et al. 2020, Maryam Omran et al. 2017). The Laplace transform is a discrete method implemented in engineering and physics to determine the output of a linear equation system. It accomplishes this by combining the input single with the unit impulse response.

In particular, the fractional derivative is important in characterizing the memory and heredity qualities of materials and processes (Saha Ray, S. 2020). The fractional order is equal to its fractional dimensions, and discontinuous media are the finest places to describe the fractional differential equations. Fractal media which are complicated arise in numerous disciplines of engineering and science. For modeling challenges for fractal mathematics and engineering on Cantorian space in fractal media, the local fractional calculus theory is crucial in this regard. One of the earlier concerns in the research of fractional differential equations is the numerical and precise solution of these equations. To date, several effective techniques have been put forth to produce accurate and numerical solutions for fractional differential equations. Nonlinear fractional partial differential equations are the most effective means of modeling the majority of nonlinear physical processes that arise in several scientific domains, including fluid dynamics, mathematical biology, solid state physics, optical fibers, plasma physics, and chemical kinetics.

When such circumstances arise, convolution becomes a multiplication operation when the computation is performed using the Laplace transform, which is more tractable to solve due to its algebraic nature. Due to the numerous advantages of the Laplace transform, electrical engineering makes extensive use of it. The Partial differential equations can be transformed by using the Laplace transform into algebraic equation. The inverse Laplace transform is then used is solve the initial Partial differential equations. The Partial differential equations and fractional Partial differential equations have both been solved utilizing this Laplace transform (Lokenath Debnath 2015, A. K.Thakur et al. 2018).

In brief the triple Laplace transform plays the role of an extension for Laplace transform in such work (Shailesh A. Bhanotar et al. 2021, Belgacem Fethi Bin Muhammad et al. 2021). In the next steps the conformable triple Laplace transforms will be adopted for the whole study. The following is a breakdown of how this article is structured. The preliminary concept along with some properties of conformable triple Laplace transform has been studied in the next section. The conformable triple Laplace transform for some basic functions are introduced in Section 3. In Section 4, an application of the operator proposed in this paper is presented for solving a few kind of third-order PDEs and the Mboctara equation has also been transformed using the conformable triple Laplace Transform. Finally, the conclusive remarks are presented for our paper.

2. SOME DEFINITIONS AND THEREMS

In this section, we give some definitions and theorems which illustrate the basic properties of Conformable triple Laplace transform

Definition 2.1: Let $f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)$ be a piecewise continuous function on the interval $[0,\infty)\times[0,\infty)\times[0,\infty)$ having the exponential order, consider for some $a,b,c\in\mathcal{R}$, $sup\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}>0$, $\exp\left(\frac{\left|f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)\right|}{a\frac{x^\alpha}{\alpha}+b\frac{y^\beta}{\beta}+c\frac{t^\delta}{\delta}}\right)<\infty$, then the conformable triple Laplace transform is denoted by $\mathcal{L}_x^\alpha\mathcal{L}_y^\beta\mathcal{L}_t^\delta\left[f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)\right]$ and defined as the following formula :

$$\mathcal{L}_x^{\alpha}\mathcal{L}_y^{\beta}\mathcal{L}_t^{\delta}\left[f\left(\frac{x^{\alpha}}{\alpha},\frac{y^{\beta}}{\beta},\frac{t^{\delta}}{\delta}\right)\right]= \int_0^{\infty}\int_0^{\infty}\int_0^{\infty} e^{-p\frac{x^{\alpha}}{\alpha}-s\frac{y^{\beta}}{\beta}-k\frac{t^{\delta}}{\delta}} f\left(\frac{x^{\alpha}}{\alpha},\frac{y^{\beta}}{\beta},\frac{t^{\delta}}{\delta}\right)x^{\alpha-1}y^{\beta-1}t^{\delta-1}\,dt^{\delta}dy^{\beta}dx^{\alpha} = F(p,s,k). \quad (1)$$

Where $p,s,k \in \mathbb{C}, 0<\alpha,\beta,\delta\leq 1$ and the integrals are in the sense of conformable fractional integral.

(Alemayehu T. Deresse, 2022) the conformable inverse triple Laplace transform, abbreviated by $f\left(\frac{x^{\alpha}}{\alpha},\frac{y^{\beta}}{\beta},\frac{t^{\delta}}{\delta}\right)$, is defined as follows:

$$f\left(\frac{x^{\alpha}}{\alpha},\frac{y^{\beta}}{\beta},\frac{t^{\delta}}{\delta}\right)= \mathcal{L}_p^{-1}\mathcal{L}_s^{-1}\mathcal{L}_k^{-1}\left[F_{\alpha,\beta,\delta}(p,s,k)\right]$$

$$=\frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty} e^{p\frac{x^{\alpha}}{\alpha}}\left[\frac{1}{2\pi i}\int_{\beta-i\infty}^{\beta+i\infty} e^{s\frac{y^{\beta}}{\beta}}\left[\frac{1}{2\pi i}\int_{\delta-i\infty}^{\delta+i\infty} e^{k\frac{t^{\delta}}{\delta}}F_{\alpha,\beta,\delta}(p,s,k)\,dk\right]dp\right]ds \quad (2).$$

Definition 2.2: Consider a function f defined as $f\left(\frac{x^{\alpha}}{\alpha},\frac{y^{\beta}}{\beta},\frac{t^{\delta}}{\delta}\right)$. The single conformable Laplace transform of f with respect to $\frac{x^{\alpha}}{\alpha}$ is defined by

$$\mathcal{L}_x^{\alpha}\left[f\left(\frac{x^{\alpha}}{\alpha},\frac{y^{\beta}}{\beta},\frac{t^{\delta}}{\delta}\right)\right]=\int_0^{\infty} e^{-p\frac{x^{\alpha}}{\alpha}} f\left(\frac{x^{\alpha}}{\alpha},\frac{y^{\beta}}{\beta},\frac{t^{\delta}}{\delta}\right)x^{\alpha-1}dx^{\alpha} = F\left(p,\frac{y^{\beta}}{\beta},\frac{t^{\delta}}{\delta}\right), \quad (3)$$

where the subscript $\frac{x^{\alpha}}{\alpha}$ on $\mathcal{L}_x^{\alpha}$ shows that for which variable the conformable Laplace transform will be applied.

Congruently the conformable Laplace transform of the same function $f\left(\frac{x^{\alpha}}{\alpha},\frac{y^{\beta}}{\beta},\frac{t^{\delta}}{\delta}\right)$ with respect to variable $\frac{y^{\beta}}{\beta}$ and $\frac{t^{\delta}}{\delta}$ can be given respectively by

$$\mathcal{L}_y^{\beta}\left[f\left(\frac{x^{\alpha}}{\alpha},\frac{y^{\beta}}{\beta},\frac{t^{\delta}}{\delta}\right)\right]=\int_0^{\infty} e^{-s\frac{y^{\beta}}{\beta}} f\left(\frac{x^{\alpha}}{\alpha},\frac{y^{\beta}}{\beta},\frac{t^{\delta}}{\delta}\right)y^{\beta-1}dy^{\beta} = F\left(\frac{x^{\alpha}}{\alpha},s,\frac{t^{\delta}}{\delta}\right). \quad (4)$$

$$\mathcal{L}_t^\delta\left[f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)\right]=\int_0^\infty e^{-k\frac{t^\delta}{\delta}}f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)t^{\delta-1}dt^\delta=F\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},k\right). \quad (5)$$

Definition 2.3: Consider a function f defined as $f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)$. The double conformable Laplace transform of f with respect to $\frac{x^\alpha}{\alpha}$ and $\frac{y^\beta}{\beta}$ is defined by

$$\mathcal{L}_x^\alpha\mathcal{L}_y^\beta\left[f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)\right]=\int_0^\infty\int_0^\infty e^{-p\frac{x^\alpha}{\alpha}-s\frac{y^\beta}{\beta}}f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)x^{\alpha-1}y^{\beta-1}\,dy^\beta dx^\alpha=F\left(p,s,\frac{t^\delta}{\delta}\right), \quad (6)$$

where the subscript $\frac{x^\alpha}{\alpha}$ and $\frac{y^\beta}{\beta}$ on $\mathcal{L}_x^\alpha\mathcal{L}_y^\beta$ shows that for which variable the conformable double Laplace transform will be applied.

Congruently, the conformable double Laplace transform of the same function $f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)$ with respect to variable $\frac{x^\alpha}{\alpha},\frac{t^\delta}{\delta}$ and $\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}$ can be given respectively by

$$\mathcal{L}_x^\alpha\mathcal{L}_t^\delta\left[f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)\right]=\int_0^\infty\int_0^\infty e^{-p\frac{x^\alpha}{\alpha}-k\frac{t^\delta}{\delta}}f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)x^{\alpha-1}t^{\delta-1}\,dt^\delta dx^\alpha=F\left(p,\frac{y^\beta}{\beta},k\right). \quad (7)$$

$$\mathcal{L}_y^\beta\mathcal{L}_t^\delta\left[f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)\right]=\int_0^\infty\int_0^\infty e^{-s\frac{y^\beta}{\beta}-k\frac{t^\delta}{\delta}}f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)y^{\beta-1}t^{\delta-1}dt^\delta dy^\beta=F\left(\frac{x^\alpha}{\alpha},s,k\right). \quad (8)$$

Because of these definitions successive transformation denoted by $\mathcal{L}_x^\alpha\mathcal{L}_y^\beta\mathcal{L}_t^\delta\left[f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)\right]$ shown in the equality equation (1). Let supposed that function $f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)$ provides the sufficient conditions (Love 1970) (E. R. Love 2009) the order of transformation can be changed, then

$$\int_0^\infty\int_0^\infty\int_0^\infty e^{-p\frac{x^\alpha}{\alpha}-s\frac{y^\beta}{\beta}-k\frac{t^\delta}{\delta}} f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right) x^{\alpha-1}y^{\beta-1}t^{\delta-1}dt^\delta dy^\beta dx^\alpha$$
$$=\int_0^\infty\int_0^\infty\int_0^\infty e^{-s\frac{y^\beta}{\beta}-k\frac{t^\delta}{\delta}-p\frac{x^\alpha}{\alpha}} f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right) y^{\beta-1}t^{\delta-1}x^{\alpha-1}dx^\alpha dt^\delta dy^\beta$$
$$=\int_0^\infty\int_0^\infty\int_0^\infty e^{-k\frac{t^\delta}{\delta}-p\frac{x^\alpha}{\alpha}-s\frac{y^\beta}{\beta}} f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right) y^{\beta-1}x^{\alpha-1}t^{\delta-1}\, dy^\beta dx^\alpha dt^\delta.$$

and symbolically can be shown as

$$\mathcal{L}_x^\alpha\mathcal{L}_y^\beta\mathcal{L}_t^\delta\left[f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)\right]=\mathcal{L}_y^\beta\mathcal{L}_t^\delta\mathcal{L}_x^\alpha\left[f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)\right]=\mathcal{L}_t^\delta\mathcal{L}_x^\alpha\mathcal{L}_y^\beta\left[f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)\right]$$
$$=F(p,s,k).$$

2.1. Some properties of conformable triple Laplace transform

In order to determine further function transforms, some prove of the properties of the conformable triple Laplace transform are given in this section, $\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)$, $F(p,s,k)$:

Theorem 2.1 Let the conformable triple Laplace transform of $f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)$ and $g\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)$ are exists and $\mathcal{L}_x^\alpha\mathcal{L}_y^\beta\mathcal{L}_t^\delta\left[f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)\right]=F(p,s,k)$ and $\mathcal{L}_x^\alpha\mathcal{L}_y^\beta\mathcal{L}_t^\delta\left[g\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)\right]=G(p,s,k)$. Then for any constants a,b and c the following properties hold:

1. $\mathcal{L}_x^\alpha\mathcal{L}_y^\beta\mathcal{L}_t^\delta\left[a\,f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)+\ a\,g\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)\right]$
$$=a\,\mathcal{L}_x^\alpha\mathcal{L}_y^\beta\mathcal{L}_t^\delta\left[f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)\right]+b\ \mathcal{L}_x^\alpha\mathcal{L}_y^\beta\mathcal{L}_t^\delta\left[g\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)\right].$$
2. $\mathcal{L}_x^\alpha\mathcal{L}_y^\beta\mathcal{L}_t^\delta\left[e^{-a\frac{x^\alpha}{\alpha}-b\frac{y^\beta}{\beta}-c\frac{t^\delta}{\delta}}f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)\right]=F(p+a,s+b,k+c).$
3. $\mathcal{L}_x^\alpha\mathcal{L}_y^\beta\mathcal{L}_t^\delta\left[f\left(a\,\frac{x^\alpha}{\alpha},b\,\frac{y^\beta}{\beta},c\,\frac{t^\delta}{\delta}\right)\right]=\frac{1}{a^\alpha b^\beta c^\delta}\,F\left(\frac{p}{a},\frac{s}{b},\frac{k}{c}\right).$
4. $(-1)^{a+b+c}\mathcal{L}_x^\alpha\mathcal{L}_y^\beta\mathcal{L}_t^\delta\left[\left(\frac{x^\alpha}{\alpha}\right)^a\left(\frac{y^\beta}{\beta}\right)^b\left(\frac{t^\delta}{\delta}\right)^c f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)\right]=\frac{\partial^{a+b+c}F(p,s,k)}{\partial p^a\partial s^b\partial k^c}$

Proof:

1. By using the definition of Conformable Triple Laplace Transform the proof of 1 can be shown easily.
2. $\mathcal{L}_x^\alpha \mathcal{L}_y^\beta \mathcal{L}_t^\delta \left[e^{-a\frac{x^\alpha}{\alpha}-b\frac{y^\beta}{\beta}-c\frac{t^\delta}{\delta}} f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right) \right]$

$$= \int_0^\infty \int_0^\infty \int_0^\infty e^{-a\frac{x^\alpha}{\alpha}-b\frac{y^\beta}{\beta}-c\frac{t^\delta}{\delta}} e^{-p\frac{x^\alpha}{\alpha}-s\frac{y^\beta}{\beta}-k\frac{t^\delta}{\delta}} f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right) x^{\alpha-1} y^{\beta-1} t^{\delta-1} dt^\delta dy^\beta dx^\alpha .$$

$$= \int_0^\infty \int_0^\infty \int_0^\infty e^{-a\frac{x^\alpha}{\alpha}-b\frac{y^\beta}{\beta}-c\frac{t^\delta}{\delta}-p\frac{x^\alpha}{\alpha}-s\frac{y^\beta}{\beta}-k\frac{t^\delta}{\delta}} f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right) x^{\alpha-1} y^{\beta-1} t^{\delta-1} dt^\delta dy^\beta dx^\alpha .$$

$$= \int_0^\infty e^{-a\frac{x^\alpha}{\alpha}-p\frac{x^\alpha}{\alpha}} \left[\int_0^\infty e^{-b\frac{y^\beta}{\beta}-s\frac{y^\beta}{\beta}} \left[\int_0^\infty e^{-c\frac{t^\delta}{\delta}-k\frac{t^\delta}{\delta}} f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right) t^{\delta-1}\, dt^\delta \right] y^{\beta-1} dy^\beta \right] x^{\alpha-1} dx^\alpha \quad (9).$$

By using conformable Triple Laplace Transform definition

$$\int_0^\infty e^{-c\frac{t^\delta}{\delta}-k\frac{t^\delta}{\delta}} f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right) t^{\delta-1}\, dt^\delta = F\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},k+c\right) \quad (10).$$

Now substituting this equation (10) into the above equation (9) which gives us:

$$\int_0^\infty e^{-a\frac{x^\alpha}{\alpha}-p\frac{x^\alpha}{\alpha}} \left[\int_0^\infty e^{-b\frac{y^\beta}{\beta}-s\frac{y^\beta}{\beta}} \left[F\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},k+c\right) \right] y^{\beta-1} dy^\beta \right] x^{\alpha-1} dx^\alpha$$

$$= F(p+a, s+b, k+c).$$

3. Setting $u = a\frac{x^\alpha}{\alpha}$, $v = b\frac{y^\beta}{\beta}$ and $z = c\frac{t^\delta}{\delta}$, then the rest of the proof as follows.

$\mathcal{L}_x^\alpha \mathcal{L}_y^\beta \mathcal{L}_t^\delta \left[f\left(a\frac{x^\alpha}{\alpha}, b\frac{y^\beta}{\beta}, c\frac{t^\delta}{\delta}\right) \right]$

$$= \int_0^\infty \int_0^\infty \int_0^\infty e^{-p\frac{x^\alpha}{\alpha}-s\frac{y^\beta}{\beta}-k\frac{t^\delta}{\delta}} f\left(a\frac{x^\alpha}{\alpha}, b\frac{y^\beta}{\beta}, c\frac{t^\delta}{\delta}\right) x^{\alpha-1} y^{\beta-1} t^{\delta-1} dt^\delta dy^\beta dx^\alpha .$$

$$= \int_0^\infty e^{-p\frac{x^\alpha}{\alpha}} \left[\int_0^\infty e^{-s\frac{y^\beta}{\beta}} \left[\int_0^\infty e^{-k\frac{t^\delta}{\delta}} f\left(a\frac{x^\alpha}{\alpha}, b\frac{y^\beta}{\beta}, c\frac{t^\delta}{\delta}\right) t^{\delta-1}\, dt^\delta \right] y^{\beta-1} dy^\beta \right] x^{\alpha-1} dx^\alpha$$

$$= \frac{1}{c^\delta} \int_0^\infty e^{-p\frac{x^\alpha}{\alpha}} \left[\int_0^\infty e^{-s\frac{y^\beta}{\beta}} \left[\int_0^\infty e^{-k\frac{t^\delta}{\delta}} f\left(a\frac{x^\alpha}{\alpha}, b\frac{y^\beta}{\beta}, z\right) dz \right] y^{\beta-1} dy^\beta \right] x^{\alpha-1} dx^\alpha$$

$$= \frac{1}{c^\delta} \int_0^\infty e^{-p\frac{x^\alpha}{\alpha}} \left[\int_0^\infty e^{-s\frac{y^\beta}{\beta}} F\left(a\frac{x^\alpha}{\alpha}, b\frac{y^\beta}{\beta}, \frac{k}{c}\right) y^{\beta-1} dy^\beta \right] x^{\alpha-1} dx^\alpha$$

$$= \frac{1}{b^{\beta}c^{\delta}} \int_0^{\infty} e^{-p\frac{x^{\alpha}}{\alpha}} \left[\int_0^{\infty} e^{-s\frac{y^{\beta}}{\beta}} F\left(a\ \frac{x^{\alpha}}{\alpha}, v, \frac{k}{c}\right) dv \right] x^{\alpha-1} dx^{\alpha}$$

$$= \frac{1}{b^{\beta}c^{\delta}} \int_0^{\infty} e^{-p\frac{x^{\alpha}}{\alpha}} F\left(a\ \frac{x^{\alpha}}{\alpha}, \frac{s}{b}, \frac{k}{c}\right) x^{\alpha-1} dx^{\alpha}$$

$$= \frac{1}{a^{\alpha}b^{\beta}c^{\delta}} \int_0^{\infty} e^{-p\frac{x^{\alpha}}{\alpha}} F\left(u, \frac{s}{b}, \frac{k}{c}\right) du$$

$$= \frac{1}{a^{\alpha}b^{\beta}c^{\delta}} F\left(\frac{p}{a}, \frac{s}{b}, \frac{k}{c}\right).$$

4. The order of differentiation and integration can be changed, due to convergence properties of the improper integral included. So we can differentiate with respect to p, s, k under the integral sign. Hence,

$$\frac{\partial^{a+b+c}F(p,s,k)}{\partial p^a \partial s^b \partial k^c}$$

$$= \int_0^{\infty} \frac{\partial^a}{\partial p^a} e^{-p\frac{x^{\alpha}}{\alpha}} \left[\int_0^{\infty} \frac{\partial^b}{\partial s^b} e^{-s\frac{y^{\beta}}{\beta}} \left[\int_0^{\infty} \frac{\partial^c}{\partial k^c} e^{-k\frac{t^{\delta}}{\delta}} f\left(a\ \frac{x^{\alpha}}{\alpha}, b\ \frac{y^{\beta}}{\beta}, c\ \frac{t^{\delta}}{\delta}\right) t^{\delta-1} dt^{\delta} \right] y^{\beta-1} dy^{\beta} \right] x^{\alpha-1} dx^{\alpha}$$

$$= (-1)^c \int_0^{\infty} \frac{\partial^a}{\partial p^a} e^{-p\frac{x^{\alpha}}{\alpha}} \left[\int_0^{\infty} \frac{\partial^b}{\partial s^b} e^{-s\frac{y^{\beta}}{\beta}} \left[\int_0^{\infty} \left(\frac{t^{\delta}}{\delta}\right)^c e^{-k\frac{t^{\delta}}{\delta}} f\left(a\ \frac{x^{\alpha}}{\alpha}, b\ \frac{y^{\beta}}{\beta}, c\ \frac{t^{\delta}}{\delta}\right) t^{\delta-1} dt^{\delta} \right] y^{\beta-1} dy^{\beta} \right] x^{\alpha-1} dx^{\alpha}$$

$$= (-1)^c \int_0^{\infty} \frac{\partial^a}{\partial p^a} e^{-p\frac{x^{\alpha}}{\alpha}} \left[\int_0^{\infty} \frac{\partial^b}{\partial s^b} e^{-s\frac{y^{\beta}}{\beta}} \left[\mathcal{L}_t^{\delta} \left[\left(\frac{t^{\delta}}{\delta}\right)^c f\left(\frac{x^{\alpha}}{\alpha}, \frac{y^{\beta}}{\beta}, \frac{t^{\delta}}{\delta}\right) \right] \right] y^{\beta-1} dy^{\beta} \right] x^{\alpha-1} dx^{\alpha}$$

$$= (-1)^{b+c} \int_0^{\infty} \frac{\partial^a}{\partial p^a} e^{-p\frac{x^{\alpha}}{\alpha}} \left[\int_0^{\infty} \left(\frac{y^{\beta}}{\beta}\right)^b e^{-s\frac{y^{\beta}}{\beta}} \left[\mathcal{L}_t^{\delta} \left[\left(\frac{t^{\delta}}{\delta}\right)^c f\left(\frac{x^{\alpha}}{\alpha}, \frac{y^{\beta}}{\beta}, \frac{t^{\delta}}{\delta}\right) \right] \right] y^{\beta-1} dy^{\beta} \right] x^{\alpha-1} dx^{\alpha}$$

$$= (-1)^{b+c} \int_0^{\infty} \frac{\partial^a}{\partial p^a} e^{-p\frac{x^{\alpha}}{\alpha}} \left[\mathcal{L}_y^{\beta} \mathcal{L}_t^{\delta} \left[\left(\frac{y^{\beta}}{\beta}\right)^b \left(\frac{t^{\delta}}{\delta}\right)^c f\left(\frac{x^{\alpha}}{\alpha}, \frac{y^{\beta}}{\beta}, \frac{t^{\delta}}{\delta}\right) \right] \right] x^{\alpha-1} dx^{\alpha}$$

Repeating differentiation with respect to p, s and k, arises the following equation

$$\frac{\partial^{a+b+c}F(p,s,k)}{\partial p^a \partial s^b \partial k^c} = (-1)^{a+b+c} \mathcal{L}_x^{\alpha} \mathcal{L}_y^{\beta} \mathcal{L}_t^{\delta} \left[\left(\frac{x^{\alpha}}{\alpha}\right)^a \left(\frac{y^{\beta}}{\beta}\right)^b \left(\frac{t^{\delta}}{\delta}\right)^c f\left(\frac{x^{\alpha}}{\alpha}, \frac{y^{\beta}}{\beta}, \frac{t^{\delta}}{\delta}\right) \right].$$

Theorem 2.2. Let $f\left(\frac{x^{\alpha}}{\alpha}, \frac{y^{\beta}}{\beta}, \frac{t^{\delta}}{\delta}\right) = \hbar\left(\frac{x^{\alpha}}{\alpha}\right) g\left(\frac{y^{\beta}}{\beta}\right) j\left(\frac{t^{\delta}}{\delta}\right), \quad \frac{x^{\alpha}}{\alpha}, \frac{y^{\beta}}{\beta}, \frac{t^{\delta}}{\delta} > 0$. Then

$$\mathcal{L}_x^{\alpha} \mathcal{L}_y^{\beta} \mathcal{L}_t^{\delta} \left[f\left(\frac{x^{\alpha}}{\alpha}, \frac{y^{\beta}}{\beta}, \frac{t^{\delta}}{\delta}\right) \right] = \mathcal{L}_x \left[\hbar\left(\frac{x^{\alpha}}{\alpha}\right) \right] \mathcal{L}_y \left[g\left(\frac{y^{\beta}}{\beta}\right) \right] \mathcal{L}_t \left[j\left(\frac{t^{\delta}}{\delta}\right) \right]. \tag{11}$$

Proof:

Using the definition of Conformable Triple Laplace Transform (CTLT), one can get:

$$\mathcal{L}_x^\alpha \mathcal{L}_y^\beta \mathcal{L}_t^\delta\left[f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)\right] = \mathcal{L}_x^\alpha \mathcal{L}_y^\beta \mathcal{L}_t^\delta\left[\hbar\left(\frac{x^\alpha}{\alpha}\right)g\left(\frac{y^\beta}{\beta}\right)j\left(\frac{t^\delta}{\delta}\right)\right]$$

$$= \int_0^\infty\int_0^\infty\int_0^\infty e^{-p\frac{x^\alpha}{\alpha}-s\frac{y^\beta}{\beta}-k\frac{t^\delta}{\delta}}\left[\hbar\left(\frac{x^\alpha}{\alpha}\right)g\left(\frac{y^\beta}{\beta}\right)j\left(\frac{t^\delta}{\delta}\right)\right]x^{\alpha-1}y^{\beta-1}t^{\delta-1}dt^\delta\, dy^\beta dx^\alpha$$

$$= \int_0^\infty e^{-p\frac{x^\alpha}{\alpha}}\left[\hbar\left(\frac{x^\alpha}{\alpha}\right)\right]x^{\alpha-1}dx^\alpha \int_0^\infty e^{-s\frac{y^\beta}{\beta}}\left[g\left(\frac{y^\beta}{\beta}\right)\right]y^{\beta-1}\, dy^\beta \int_0^\infty e^{-k\frac{t^\delta}{\delta}}\left[j\left(\frac{t^\delta}{\delta}\right)\right]t^{\delta-1}\, dt^\delta. \quad (12)$$

Substituting the values of $u = \frac{x^\alpha}{\alpha}, v = \frac{y^\beta}{\beta}, z = \frac{t^\delta}{\delta},\ du = x^{\alpha-1}dx^\alpha,\ dv = y^{\beta-1}dy^\beta$ and $dz = t^{\delta-1}dt^\delta$ into Eq. (12) and simplifying, one can find

$$\boldsymbol{\mathcal{L}_x^\alpha \mathcal{L}_y^\beta \mathcal{L}_t^\delta\left[\hbar\left(\frac{x^\alpha}{\alpha}\right)g\left(\frac{y^\beta}{\beta}\right)j\left(\frac{t^\delta}{\delta}\right)\right] = \int_0^\infty e^{-pu}[\hbar(u)]du \int_0^\infty e^{-sv}[g(v)]\, dv \int_0^\infty e^{-kz}[j(z)]\, dz.}$$
$$\boldsymbol{= \mathcal{L}_x[\hbar(x)]\, \mathcal{L}_y[g(\mathrm{y})]\mathcal{L}_t[j(\mathrm{t})].}$$

3. CONFORMABLE TRIPLE LAPLACE TRANSFORM OF SOME BASIC FUNCTIONS

In the following arguments, I introduce the conformable triple Laplace transform for some basic functions

i. Let $f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right) = 1$. Then

$$\mathcal{L}_x^\alpha \mathcal{L}_y^\beta \mathcal{L}_t^\delta[1] = \int_0^\infty\int_0^\infty\int_0^\infty e^{-p\frac{x^\alpha}{\alpha}-s\frac{y^\beta}{\beta}-k\frac{t^\delta}{\delta}}x^{\alpha-1}y^{\beta-1}t^{\delta-1}dt^\delta dy^\beta dx^\alpha .$$

From theorem 2.2 and definition of Laplace transform, one can get

$$\mathcal{L}_x^\alpha \mathcal{L}_y^\beta \mathcal{L}_t^\delta[1] = \mathcal{L}_x[\hbar(1)]\, \mathcal{L}_y[g(1)]\mathcal{L}_t[j(1)] = \frac{1}{psk}.$$

ii. Let $f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right) = \left(\frac{x^\alpha}{\alpha}\right)^n\left(\frac{y^\beta}{\beta}\right)^m\left(\frac{t^\delta}{\delta}\right)^r$. Then

$$\mathcal{L}_x^{\alpha}\mathcal{L}_y^{\beta}\mathcal{L}_t^{\delta}\left[\left(\frac{x^{\alpha}}{\alpha}\right)^n\left(\frac{y^{\beta}}{\beta}\right)^m\left(\frac{t^{\delta}}{\delta}\right)^r\right] = \int_0^{\infty}\int_0^{\infty}\int_0^{\infty} e^{-p\frac{x^{\alpha}}{\alpha}-s\frac{y^{\beta}}{\beta}-k\frac{t^{\delta}}{\delta}}\left[\left(\frac{x^{\alpha}}{\alpha}\right)^n\left(\frac{y^{\beta}}{\beta}\right)^m\left(\frac{t^{\delta}}{\delta}\right)^r\right] x^{\alpha-1}y^{\beta-1}t^{\delta-1}dt^{\delta}dy^{\beta}dx^{\alpha}\ .$$

From theorem 2.2 and definition of Laplace transform, one can get:

$$\mathcal{L}_x^{\alpha}\mathcal{L}_y^{\beta}\mathcal{L}_t^{\delta}\left[\left(\frac{x^{\alpha}}{\alpha}\right)^n\left(\frac{y^{\beta}}{\beta}\right)^m\left(\frac{t^{\delta}}{\delta}\right)^r\right] = \mathcal{L}_x[x^n]\,\mathcal{L}_y[y^m]\mathcal{L}_t[t^r] = \frac{n!\,m!\,\mathrm{r}!}{p^{n+1}s^{m+1}k^{r+1}}.$$

iii. Let $f\left(\frac{x^{\alpha}}{\alpha},\frac{y^{\beta}}{\beta},\frac{t^{\delta}}{\delta}\right) = e^{m\frac{x^{\alpha}}{\alpha}+n\frac{y^{\beta}}{\beta}+r\frac{t^{\delta}}{\delta}}$.

Then

$$\mathcal{L}_x^{\alpha}\mathcal{L}_y^{\beta}\mathcal{L}_t^{\delta}\left[e^{m\frac{x^{\alpha}}{\alpha}+n\frac{y^{\beta}}{\beta}+r\frac{t^{\delta}}{\delta}}\right] = \int_0^{\infty}\int_0^{\infty}\int_0^{\infty} e^{-p\frac{x^{\alpha}}{\alpha}-s\frac{y^{\beta}}{\beta}-k\frac{t^{\delta}}{\delta}}\left[e^{m\frac{x^{\alpha}}{\alpha}+n\frac{y^{\beta}}{\beta}+r\frac{t^{\delta}}{\delta}}\right] x^{\alpha-1}y^{\beta-1}t^{\delta-1}\,dt^{\delta}dy^{\beta}dx^{\alpha}.$$

From theorem 2.2 and definition of Laplace transform, one can get:

$$\mathcal{L}_x^{\alpha}\mathcal{L}_y^{\beta}\mathcal{L}_t^{\delta}\left[e^{m\frac{x^{\alpha}}{\alpha}+n\frac{y^{\beta}}{\beta}+r\frac{t^{\delta}}{\delta}}\right] = \mathcal{L}_x[e^{mx}]\,\mathcal{L}_y[e^{ny}]\mathcal{L}_t[e^{rt}] = \frac{1}{(p-m)(s-n)(k-r)}.$$

iv. Let $f(\frac{x^{\alpha}}{\alpha},\frac{y^{\beta}}{\beta},\frac{t^{\delta}}{\delta}) = \sin\left(\frac{x^{\alpha}}{\alpha}\right)\sin\left(\frac{y^{\beta}}{\beta}\right)\sin\left(\frac{t^{\delta}}{\delta}\right)$. Then

$$\mathcal{L}_x^{\alpha}\mathcal{L}_y^{\beta}\mathcal{L}_t^{\delta}\left[\sin\left(\frac{x^{\alpha}}{\alpha}\right)\sin\left(\frac{y^{\beta}}{\beta}\right)\sin\left(\frac{t^{\delta}}{\delta}\right)\right] = \int_0^{\infty}\int_0^{\infty}\int_0^{\infty} e^{-p\frac{x^{\alpha}}{\alpha}-s\frac{y^{\beta}}{\beta}-k\frac{t^{\delta}}{\delta}}\left[\sin\left(\frac{x^{\alpha}}{\alpha}\right)\sin\left(\frac{y^{\beta}}{\beta}\right)\sin\left(\frac{t^{\delta}}{\delta}\right)\right] x^{\alpha-1}y^{\beta-1}t^{\delta-1}\,dt^{\delta}dy^{\beta}dx^{\alpha}\ .$$

From theorem 2.2 and definition of Laplace transform, one can get:

$$\mathcal{L}_x^{\alpha}\mathcal{L}_y^{\beta}\mathcal{L}_t^{\delta}\left[\sin\left(\frac{x^{\alpha}}{\alpha}\right)\sin\left(\frac{y^{\beta}}{\beta}\right)\sin\left(\frac{t^{\delta}}{\delta}\right)\right]=\mathcal{L}_x\left[\sin\left(\frac{x^{\alpha}}{\alpha}\right)\right]\mathcal{L}_y\left[\sin\left(\frac{y^{\beta}}{\beta}\right)\right]\mathcal{L}_t\left[\sin\left(\frac{t^{\delta}}{\delta}\right)\right]$$
$$=\frac{1}{(p^2+1)(s^2+1)(k^2+1)}.$$

v. Let $f(\frac{x^{\alpha}}{\alpha},\frac{y^{\beta}}{\beta},\frac{t^{\delta}}{\delta})=\cos\left(\frac{x^{\alpha}}{\alpha}\right)\cos\left(\frac{y^{\beta}}{\beta}\right)\cos\left(\frac{t^{\delta}}{\delta}\right)$. Then

$$\mathcal{L}_x^{\alpha}\mathcal{L}_y^{\beta}\mathcal{L}_t^{\delta}\left[\cos\left(\frac{x^{\alpha}}{\alpha}\right)\cos\left(\frac{y^{\beta}}{\beta}\right)\cos\left(\frac{t^{\delta}}{\delta}\right)\right]$$
$$=\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}e^{-p\frac{x^{\alpha}}{\alpha}-s\frac{y^{\beta}}{\beta}-k\frac{t^{\delta}}{\delta}}\left[\cos\left(\frac{x^{\alpha}}{\alpha}\right)\cos\left(\frac{y^{\beta}}{\beta}\right)\cos\left(\frac{t^{\delta}}{\delta}\right)\right]x^{\alpha-1}y^{\beta-1}t^{\delta-1}\,dt^{\delta}dy^{\beta}dx^{\alpha}.$$

From theorem 2.2 and definition of Laplace transform, one can get:

$$\mathcal{L}_x^{\alpha}\mathcal{L}_y^{\beta}\mathcal{L}_t^{\delta}\left[\cos\left(\frac{x^{\alpha}}{\alpha}\right)\cos\left(\frac{y^{\beta}}{\beta}\right)\cos\left(\frac{t^{\delta}}{\delta}\right)\right]=\mathcal{L}_x\left[\cos\left(\frac{x^{\alpha}}{\alpha}\right)\right]\mathcal{L}_y\left[\cos\left(\frac{y^{\beta}}{\beta}\right)\right]\mathcal{L}_t\left[\cos\left(\frac{t^{\delta}}{\delta}\right)\right]$$
$$=\frac{psk}{(p^2+1)(s^2+1)(k^2+1)}.$$

Theorem 3.1. Assuming that the continuous function $f\left(\frac{x^{\alpha}}{\alpha},\frac{y^{\beta}}{\beta},\frac{t^{\delta}}{\delta}\right)$ is conformable triple Laplace transformable, then

i. $\mathcal{L}_x^{\alpha}\mathcal{L}_y^{\beta}\mathcal{L}_t^{\delta}\left[\frac{\partial}{\partial x^{\alpha}}f\left(\frac{x^{\alpha}}{\alpha},\frac{y^{\beta}}{\beta},\frac{t^{\delta}}{\delta}\right)\right]=p\,F(p,s,k)-F(0,s,k).$

ii. $\mathcal{L}_x^{\alpha}\mathcal{L}_y^{\beta}\mathcal{L}_t^{\delta}\left[\frac{\partial^2}{\partial x^{\alpha}\partial y^{\beta}}f\left(\frac{x^{\alpha}}{\alpha},\frac{y^{\beta}}{\beta},\frac{t^{\delta}}{\delta}\right)\right]=ps\,F(p,s,k)-pF(p,0,k)-sF(0,s,k)-F(0,0,k).$

iii. $\mathcal{L}_x^{\alpha}\mathcal{L}_y^{\beta}\mathcal{L}_t^{\delta}\left[\frac{\partial^3}{\partial x^{\alpha}\partial y^{\beta}\partial t^{\delta}}f\left(\frac{x^{\alpha}}{\alpha},\frac{y^{\beta}}{\beta},\frac{t^{\delta}}{\delta}\right)\right]=psk\,F(p,s,k)-psF(p,s,0)-skF(p,0,k)-$
$pkF(0,s,k)-F(0,0,0)+pF(p,0,0)+kF(0,0,k)+sF(0,s,0).$

iv. $\mathcal{L}_x^{\alpha}\mathcal{L}_y^{\beta}\mathcal{L}_t^{\delta}\left[\frac{\partial^3}{\partial x^{2\alpha}\partial y^{\beta}}f\left(\frac{x^{\alpha}}{\alpha},\frac{y^{\beta}}{\beta},\frac{t^{\delta}}{\delta}\right)\right]=\mathrm{s}p^2\,F(p,s,k)-psF(0,s,k)-\frac{\partial F(0,s,k)}{\partial x^{\alpha}}-p^2F(p,0,k)+$
$pF(0,0,k)+\frac{\partial F(0,0,k)}{\partial x^{\alpha}}.$

v. $\mathcal{L}_x^{\alpha}\mathcal{L}_y^{\beta}\mathcal{L}_t^{\delta}\left[\frac{\partial^3}{\partial x^{3\alpha}}f\left(\frac{x^{\alpha}}{\alpha},\frac{y^{\beta}}{\beta},\frac{t^{\delta}}{\delta}\right)\right]=p^3F(p,s,k)-p^2F(0,s,k)-p\frac{\partial F(0,s,k)}{\partial x^{\alpha}}-\frac{\partial^2F(0,s,k)}{\partial x^{2\alpha}}.$

Proof:

i. Using the definition of conformable triple Laplace transform for $\frac{\partial}{\partial x^{\alpha}} f\left(\frac{x^{\alpha}}{\alpha}, \frac{y^{\beta}}{\beta}, \frac{t^{\delta}}{\delta}\right)$, one can have

$$\mathcal{L}_x^{\alpha}\mathcal{L}_y^{\beta}\mathcal{L}_t^{\delta}\left[\frac{\partial}{\partial x^{\alpha}} f\left(\frac{x^{\alpha}}{\alpha}, \frac{y^{\beta}}{\beta}, \frac{t^{\delta}}{\delta}\right)\right]$$

$$= \int_0^{\infty}\int_0^{\infty}\int_0^{\infty} e^{-p\frac{x^{\alpha}}{\alpha}-s\frac{y^{\beta}}{\beta}-k\frac{t^{\delta}}{\delta}}\left[\frac{\partial}{\partial x^{\alpha}} f\left(\frac{x^{\alpha}}{\alpha}, \frac{y^{\beta}}{\beta}, \frac{t^{\delta}}{\delta}\right)\right] x^{\alpha-1}y^{\beta-1}t^{\delta-1}dt^{\delta}dy^{\beta}dx^{\alpha}. \quad (13)$$

$$= \int_0^{\infty}\int_0^{\infty} e^{-s\frac{y^{\beta}}{\beta}-k\frac{t^{\delta}}{\delta}}\left\{\int_0^{\infty} e^{-p\frac{x^{\alpha}}{\alpha}}\left[\frac{\partial}{\partial x^{\alpha}} f\left(\frac{x^{\alpha}}{\alpha}, \frac{y^{\beta}}{\beta}, \frac{t^{\delta}}{\delta}\right)\right] x^{\alpha-1}dx^{\alpha}\right\} y^{\beta-1}t^{\delta-1}dt^{\delta}dy^{\beta}.$$

The integral inside bracket given by

$$\int_0^{\infty} e^{-p\frac{x^{\alpha}}{\alpha}}\left[\frac{\partial}{\partial x^{\alpha}} f\left(\frac{x^{\alpha}}{\alpha}, \frac{y^{\beta}}{\beta}, \frac{t^{\delta}}{\delta}\right)\right] x^{\alpha-1}dx^{\alpha} = p\,F\left(p, \frac{y^{\beta}}{\beta}, \frac{t^{\delta}}{\delta}\right) - F\left(0, \frac{y^{\beta}}{\beta}, \frac{t^{\delta}}{\delta}\right) \quad (14)$$

By substituting equation (14) into equation (11), one can obtain

$$\mathcal{L}_x^{\alpha}\mathcal{L}_y^{\beta}\mathcal{L}_t^{\delta}\left[\frac{\partial}{\partial x^{\alpha}} f\left(\frac{x^{\alpha}}{\alpha}, \frac{y^{\beta}}{\beta}, \frac{t^{\delta}}{\delta}\right)\right] = p\,F(p,s,k) - F(0,s,k). \quad (15)$$

In the same manner, one can easily prove ii and iii .

The conformable triple Laplace transformable of $\frac{\partial^2}{\partial x^{\alpha}\partial y^{\beta}} f\left(\frac{x^{\alpha}}{\alpha}, \frac{y^{\beta}}{\beta}, \frac{t^{\delta}}{\delta}\right)$ and $\frac{\partial^3}{\partial x^{\alpha}\partial y^{\beta}\partial t^{\delta}} f\left(\frac{x^{\alpha}}{\alpha}, \frac{y^{\beta}}{\beta}, \frac{t^{\delta}}{\delta}\right)$ can be obtained.

iv. Using the definition of conformable triple Laplace transform for $\frac{\partial^3}{\partial x^{2\alpha}\partial y^{\beta}} f\left(\frac{x^{\alpha}}{\alpha}, \frac{y^{\beta}}{\beta}, \frac{t^{\delta}}{\delta}\right)$, one can have

$$\mathcal{L}_x^{\alpha}\mathcal{L}_y^{\beta}\mathcal{L}_t^{\delta}\left[\frac{\partial^3}{\partial x^{2\alpha}\partial y^{\beta}} f\left(\frac{x^{\alpha}}{\alpha}, \frac{y^{\beta}}{\beta}, \frac{t^{\delta}}{\delta}\right)\right]$$

$$= \int_0^{\infty}\int_0^{\infty}\int_0^{\infty} e^{-p\frac{x^{\alpha}}{\alpha}-s\frac{y^{\beta}}{\beta}-k\frac{t^{\delta}}{\delta}}\left[\frac{\partial^3}{\partial x^{2\alpha}\partial y^{\beta}} f\left(\frac{x^{\alpha}}{\alpha}, \frac{y^{\beta}}{\beta}, \frac{t^{\delta}}{\delta}\right)\right] x^{\alpha-1}y^{\beta-1}t^{\delta-1}dt^{\delta}dy^{\beta}dx^{\alpha}. \quad (16)$$

$$= \int_0^\infty e^{-k\frac{t^\delta}{\delta}} \left[\int_0^\infty e^{-s\frac{y^\beta}{\beta}} \left[\int_0^\infty e^{-p\frac{x^\alpha}{\alpha}} \left[\frac{\partial^2}{\partial x^{2\alpha}} f\left(\frac{x^\alpha}{\alpha}, \frac{y^\beta}{\beta}, \frac{t^\delta}{\delta}\right) \right] x^{\alpha-1} dx^\alpha \right] y^{\beta-1} dy^\beta \right] t^{\delta-1} dt^\delta.$$

The integral inside bracket given by

$$\int_0^\infty e^{-p\frac{x^\alpha}{\alpha}} \left[\frac{\partial^2}{\partial x^{2\alpha}} f\left(\frac{x^\alpha}{\alpha}, \frac{y^\beta}{\beta}, \frac{t^\delta}{\delta}\right) \right] x^{\alpha-1} dx^\alpha$$

$$= p^2 F\left(p, \frac{y^\beta}{\beta}, \frac{t^\delta}{\delta}\right) - pF\left(0, \frac{y^\beta}{\beta}, \frac{t^\delta}{\delta}\right) - \frac{\partial F\left(0, \frac{y^\beta}{\beta}, \frac{t^\delta}{\delta}\right)}{\partial x^\alpha} \tag{17}$$

By substituting equation (17) into equation (16), one can obtain

$$\int_0^\infty e^{-k\frac{t^\delta}{\delta}} \left[\int_0^\infty e^{-s\frac{y^\beta}{\beta}} \left[\int_0^\infty e^{-p\frac{x^\alpha}{\alpha}} \left[\frac{\partial^2}{\partial x^{2\alpha}} f\left(\frac{x^\alpha}{\alpha}, \frac{y^\beta}{\beta}, \frac{t^\delta}{\delta}\right) \right] x^{\alpha-1} dx^\alpha \right] y^{\beta-1} dy^\beta \right] t^{\delta-1} dt^\delta$$

$$= sp^2 F(p,s,k) - psF\left(0, s, \frac{t^\delta}{\delta}\right) - \frac{\partial F\left(0, s, \frac{t^\delta}{\delta}\right)}{\partial x^\alpha} - p^2 F\left(p, 0, \frac{t^\delta}{\delta}\right) + pF\left(0,0, \frac{t^\delta}{\delta}\right)$$

$$+ \frac{\partial F\left(0,0, \frac{t^\delta}{\delta}\right)}{\partial x^\alpha} \,. \tag{18}$$

By substituting equation (18) into equation (17), one can obtain:

$$\mathcal{L}_x^\alpha \mathcal{L}_y^\beta \mathcal{L}_t^\delta \left[\frac{\partial^3}{\partial x^{2\alpha} \partial y^\beta} f\left(\frac{x^\alpha}{\alpha}, \frac{y^\beta}{\beta}, \frac{t^\delta}{\delta}\right) \right]$$

$$= sp^2 F(p,s,k) - psF(0,s,k) - \frac{\partial F(0,s,k)}{\partial x^\alpha} - p^2 F(p,0,k) + pF(0,0,k)$$

$$+ \frac{\partial F(0,0,k)}{\partial x^\alpha} \,. \tag{19}$$

In the same manner, the conformable triple Laplace transform of $\frac{\partial^3}{\partial x^{3\alpha}} f\left(\frac{x^\alpha}{\alpha}, \frac{y^\beta}{\beta}, \frac{t^\delta}{\delta}\right)$ can be obtained.

4. APPLICATION TO THIRD-ORDER PARTIAL DIFFERENTIAL EQUATION

In this section, I present the application of this operator for solving some kind of third-order partial differential equations.

Example 1. Consider the following third-order partial differential equation (Abdon Atangana 2013).

$$\frac{\partial^3}{\partial x^\alpha \partial y^\beta \partial t^\delta} f\left(\frac{x^\alpha}{\alpha}, \frac{y^\beta}{\beta}, \frac{t^\delta}{\delta}\right) + f\left(\frac{x^\alpha}{\alpha}, \frac{y^\beta}{\beta}, \frac{t^\delta}{\delta}\right) = 0\,, \quad 0 < \alpha, \beta, \delta \leq 1 \tag{20}$$

Equation (20) is commonly referred to as the Mboctara equation and the initial and boundaries conditions associated with this equation are given as follows:

$$\begin{array}{ll} f\left(\frac{x^\alpha}{\alpha}, \frac{y^\beta}{\beta}, 0\right) = e^{\frac{x^\alpha}{\alpha} + \frac{y^\beta}{\beta}}\,, & f\left(\frac{x^\alpha}{\alpha}, 0, \frac{t^\delta}{\delta}\right) = e^{\frac{x^\alpha}{\alpha} - \frac{t^\delta}{\delta}} \\ f\left(0, \frac{y^\beta}{\beta}, \frac{t^\delta}{\delta}\right) = e^{\frac{y^\beta}{\beta} - \frac{t^\delta}{\delta}}\,, & f\left(\frac{x^\alpha}{\alpha}, \frac{y^\beta}{\beta}, 1\right) = e^{\frac{x^\alpha}{\alpha} + \frac{y^\beta}{\beta} - 1} \end{array}. \tag{21}$$

Utilizing the conformable triple Laplace transform in (20), we have

$$\begin{aligned} psk\, F(p,s,k) - psF(p,s,0) - pkF(p,0,k) - skF(0,s,k) + pF(p,0,0) \\ + sF(0,s,0) + kF(0,0,k) - F(0,0,0) + F(p,s,k) = 0 \end{aligned} \tag{22}$$

Then,

$$\begin{aligned} F(p,s,k)[1+psk] = {} & psF(p,s,0) + pkF(p,0,k) + skF(0,s,k) \\ & - pF(p,0,0) - sF(0,s,0) + F(0,0,0) - kF(0,0,k) \end{aligned}$$

$$F(p,s,k) = \frac{\begin{array}{c} psF(p,s,0) + pkF(p,0,k) + skF(0,s,k) - pF(p,0,0) \\ -sF(0,s,0) + F(0,0,0) - kF(0,0,k) \end{array}}{1+psk}$$

After substituting the boundary conditions and initial conditions, one can get:

$$F(p,s,k) = \frac{1+psk}{(1+psk)(p-1)(s-1)(k+1)} = \frac{1}{(p-1)(s-1)(k+1)} \tag{23}$$

Utilizing the inverse conformable triple Laplace transform in (23), we have:

$$f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right) = \mathcal{L}_x^{-1}\mathcal{L}_y^{-1}\mathcal{L}_t^{-1}\left[\frac{1}{(p-1)(s-1)(k+1)}\right]$$

$$f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right) = e^{\frac{x^\alpha}{\alpha}+\frac{y^\beta}{\beta}-\frac{t^\delta}{\delta}} \tag{24}$$

Example 2. Let us consider the following nonhomogeneous Mboctara equation (Abdon Atangana 2013).

$$\frac{\partial^3}{\partial x^\alpha \partial y^\beta \partial t^\delta} f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right) + f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right) = -e^{\frac{x^\alpha}{\alpha}-2\frac{y^\beta}{\beta}+\frac{t^\delta}{\delta}}, \quad 0<\alpha,\beta,\delta \le 1 \tag{25}$$

The following boundary and initial conditions will governed:

$$f\left(\frac{x^\alpha}{\alpha},0,0\right) = e^{\frac{x^\alpha}{\alpha}}, \quad f\left(\frac{x^\alpha}{\alpha},0,\frac{t^\delta}{\delta}\right) = e^{\frac{x^\alpha}{\alpha}+\frac{t^\delta}{\delta}}$$
$$f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},0\right) = e^{\frac{x^\alpha}{\alpha}-2\frac{y^\beta}{\beta}}, \quad f(0,0,0) = 1, \; f\left(\frac{x^\alpha}{\alpha},0.5,\frac{t^\delta}{\delta}\right) = e^{\frac{x^\alpha}{\alpha}+\frac{t^\delta}{\delta}-1}. \tag{26}$$

The following can be obtained by applying the conformable triple Laplace transform to both sides of equation (25).

$$\begin{gathered} psk\,F(p,s,k) - psF(p,s,0) - pkF(p,0,k) - skF(0,s,k) \\ + pF(p,0,0) + sF(0,s,0) + kF(0,0,k) - F(0,0,0) + F(p,s,k) \\ = -\frac{1}{(p-1)(s+2)(k-1)} \end{gathered} \tag{27}$$

Then

$$\begin{aligned} &F(p,s,k)[1+psk] \\ &= psF(p,s,0) + pkF(p,0,k) + skF(0,s,k) - pF(p,0,0) \\ &-sF(0,s,0) - kF(0,0,k) + F(0,0,0) - \frac{1}{(p-1)(s+2)(k-1)} \end{aligned}$$

After substituting the boundary conditions and initial conditions, one can get:

$$F(p,s,k)= \frac{1+psk}{(1+psk)(p-1)(s+2)(k-1)} = \frac{1}{(p-1)(s+2)(k-1)} \tag{28}$$

Utilizing the inverse conformable triple Laplace transform in (28), we have:

$$f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)= \mathcal{L}_x^{-1}\mathcal{L}_y^{-1}\mathcal{L}_t^{-1}\left[\frac{1}{(p-1)(s+2)(k-1)}\right]$$

$$f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right) = e^{\frac{x^\alpha}{\alpha}-2\frac{y^\beta}{\beta}+\frac{t^\delta}{\delta}} \tag{29}$$

Example 3. Let us consider the following nonhomogeneous Mboctara equation (Abdon Atangana 2013).

$$\begin{gathered}\frac{\partial^3}{\partial x^\alpha \partial y^\beta \partial t^\delta} f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right)+f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right) \\ = \cos\left(\frac{x^\alpha}{\alpha}\right)\cos\left(\frac{y^\beta}{\beta}\right)\cos\left(-\frac{t^\delta}{\delta}\right)-\sin\left(\frac{x^\alpha}{\alpha}\right)\sin\left(\frac{y^\beta}{\beta}\right)\sin\left(-\frac{t^\delta}{\delta}\right)\end{gathered} \quad , \quad 0<\alpha,\beta,\delta\le 1 \tag{30}$$

The following boundary and initial conditions will governed:

$$\begin{gathered} f\left(\frac{x^\alpha}{\alpha},0,0\right)=\cos\left(\frac{x^\alpha}{\alpha}\right) \quad , \quad f\left(\frac{x^\alpha}{\alpha},0,\frac{t^\delta}{\delta}\right)= \cos\left(\frac{x^\alpha}{\alpha}\right)\cos\left(-\frac{t^\delta}{\delta}\right) \\ f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},0\right)=\cos\left(\frac{x^\alpha}{\alpha}\right)\cos\left(\frac{y^\beta}{\beta}\right) \ , \ f(0,0,0)= 1 \ , \ f\left(\frac{x^\alpha}{\alpha},\frac{\pi}{2},\frac{t^\delta}{\delta}\right)= 0 \end{gathered} , \tag{31}$$

By utilizing the conformable triple Laplace transform on both sides of equation (30), the following results can be obtained:

$$\begin{gathered} psk\,F(p,s,k)-psF(p,s,0) - pkF(p,0,k)-skF(0,s,k) \\ +pF(p,0,0)+sF(0,s,0)+kF(0,0,k)- F(0,0,0)+F(p,s,k) \\ = \frac{psk}{(p^2+1)(s^2+1)(k^2-1)} - \frac{1}{(p^2+1)(s^2+1)(k^2-1)} \end{gathered}$$

Then

$$F(p,s,k)[1+psk] = psF(p,s,0) + pkF(p,0,k) + skF(0,s,k) - pF(p,0,0) - sF(0,s,0) - kF(0,0,k) + F(0,0,0) + \frac{psk}{(p^2+1)(s^2+1)(k^2-1)} - \frac{1}{(p^2+1)(s^2+1)(k^2-1)}$$

After substituting the boundary conditions and initial conditions, one can get:

$$F(p,s,k) = \frac{psk\,(1+psk)}{(1+psk)(p^2+1)(s^2+1)(k^2-1)} = \frac{psk}{(p^2+1)(s^2+1)(k^2-1)} \tag{32}$$

By employing the inverse conformable triple Laplace transform on equation (32), the subsequent solution can be obtained:

$$f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right) = \mathcal{L}_x^{-1}\mathcal{L}_y^{-1}\mathcal{L}_t^{-1}\left[\frac{psk}{(p^2+1)(s^2+1)(k^2-1)}\right]$$

$$f\left(\frac{x^\alpha}{\alpha},\frac{y^\beta}{\beta},\frac{t^\delta}{\delta}\right) = \cos\left(\frac{x^\alpha}{\alpha}\right)\cos\left(\frac{y^\beta}{\beta}\right)\cos\left(-\frac{t^\delta}{\delta}\right). \tag{33}$$

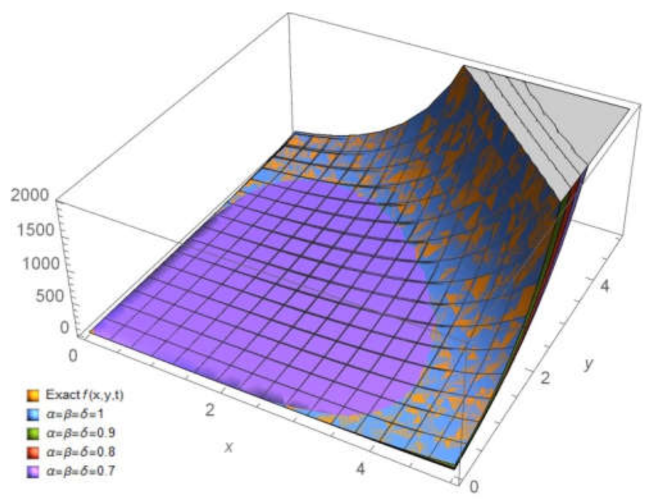

Figure 1. Numerical solution Eq (24), when

$t = 0.1\,, \alpha = \beta = \delta = \{0.9, 0.8, 0.7\}$

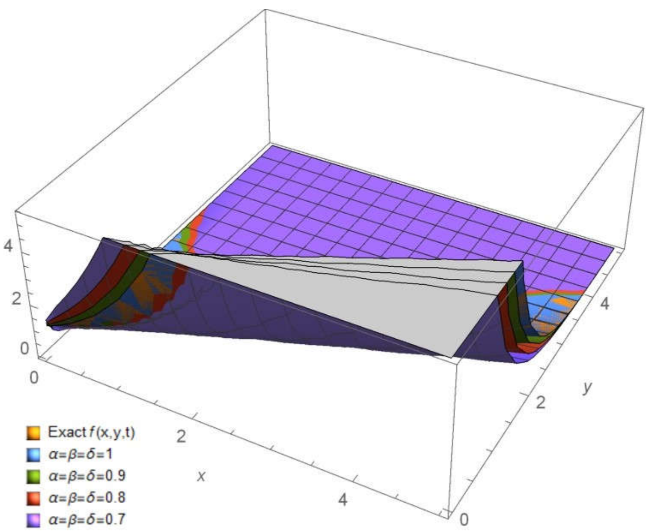

Figure 2.Numerical solution Eq (29), when

$t = 0.1\,, \alpha = \beta = \delta = \{0.9, 0.8, 0.7\}$

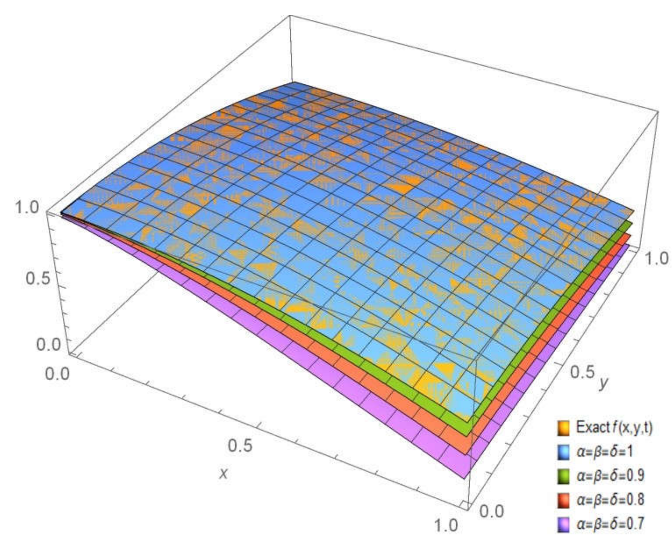

Figure 3. Numerical solution Eq (33), when

$\boldsymbol{t = 0.1\,, \alpha = \beta = \delta = \{0.9, 0.8, 0.7\}}$

Table 1. Exact and proposed method solution of example 1 at various fractional orders

t	y	x	$\alpha=\beta=\delta=0.7$	$\alpha=\beta=\delta=0.8$	$\alpha=\beta=\delta=0.9$	$\alpha=\beta=\delta=1$	Exact
		0	1	1	1	1	1
		0.1	1.32981	1.2191	1.15014	1.10517	1.10517
		0.2	1.5889	1.41189	1.29826	1.2214	1.2214
		0.3	1.84969	1.61139	1.45642	1.34986	1.34986
		0.4	2.1217	1.82314	1.62758	1.49182	1.49182
0.1	0.1	0.5	2.40943	2.0502	1.81381	1.64872	1.64872
		0.6	2.71583	2.29487	2.01699	1.82212	1.82212
		0.7	3.04328	2.55921	2.23898	2.01375	2.01375
		0.8	3.39391	2.84522	2.48171	2.22554	2.22554
		0.9	3.76973	3.1549	2.74723	2.4596	2.4596
		1	4.17273	3.49034	3.03773	2.71828	2.71828

Table 2. Exact and proposed method solution of example 2 at various fractional orders

t	y	x	$\alpha=\beta=\delta=0.7$	$\alpha=\beta=\delta=0.8$	$\alpha=\beta=\delta=0.9$	$\alpha=\beta=\delta=1$	Exact
		0	0.751986	0.820278	0.869462	0.904837	0.904837
		0.1	1	1	1	1	1
		0.2	1.19483	1.15815	1.12879	1.10517	1.10517
		0.3	1.39094	1.32179	1.2663	1.2214	1.2214
		0.4	1.59549	1.49548	1.41512	1.34986	1.34986
0.1	0.1	0.5	1.81186	1.68173	1.57704	1.49182	1.49182
		0.6	2.04227	1.88243	1.7537	1.64872	1.64872
		0.7	2.28851	2.09926	1.94671	1.82212	1.82212
		0.8	2.55217	2.33387	2.15775	2.01375	2.01375
		0.9	2.83478	2.5879	2.38861	2.22554	2.22554
		1.	3.13784	2.86305	2.64119	2.4596	2.4596

Table 3. Exact and proposed method solution of example 3 at various fractional orders

t	y	x	$\alpha=\beta=\delta=0.7$	$\alpha=\beta=\delta=0.8$	$\alpha=\beta=\delta=0.9$	$\alpha=\beta=\delta=1$	Exact
		0.	0.92093	0.961263	0.980561	0.990033	0.990033
		0.1	0.883772	0.94246	0.970983	0.985087	0.985087
		0.2	0.823953	0.904643	0.947345	0.970299	0.970299
		0.3	0.752183	0.85392	0.912066	0.945815	0.945815
		0.4	0.67244	0.793059	0.866519	0.911881	0.911881
0.1	0.1	0.5	0.587205	0.723989	0.811814	0.868836	0.868836
		0.6	0.49828	0.648254	0.748958	0.81711	0.81711
		0.7	0.407078	0.567175	0.678916	0.757219	0.757219
		0.8	0.314759	0.481928	0.60263	0.689763	0.689763
		0.9	0.2223	0.393578	0.52103	0.615415	0.615415
		1.	0.130538	0.303108	0.435041	0.534917	0.534917

5. NUMERICAL RESULT

We compare the precise solution with the second-order approximations for $\alpha=\beta=\delta=1$ in order to demonstrate the effectiveness of the conformable triple Laplace transform. The numerical solution demonstrates that the conformable triple Laplace transform systematically solves this problem successfully; We present numerical results for a range of α,β and δ in Figs. 1, 2 and 3. We can increase accuracy by using higher-order approximation solutions. The use of the conformable double Laplace transforms to address issues involving the unknown functions of three variables was challenging and frequently unsuccessful. To solve the problem, the conformable double Laplace transform was combined to create a new triple transform called the conformable triple Laplace transform. This transformation was then investigated by using properties and theorems to solve several partial differential equations(MZ Mohamed, Amjad Hamza et al. 2023). Ultimately, this work aims to construct the conformable triple Laplace transform, which may be used to convert partial differential equations into algebraic equations and solve them. The conformable triple Laplace transform provides a fast convergence to the exact solution without making any restricting assumptions on the solution, in contrast to other methods. Regretfully, nonlinear partial differential equations cannot be solved using this transform or any other integral transform. This transform is frequently used with other approaches, such as the variational technique, the differential transform method, and the homotropy method, to address this challenge.

6. CONCLUSION

In this paper, conformable triple Laplace transform was explored to solve the partial differential equations. Various definitions, characteristics, and theories were created, and their findings were presented. Some significant results were proved by using the obtained properties. Additionally, the new transform was applied to solve Mboctara equations. The Mboctara equation has also been transformed using the conformable triple Laplace Transform. The Mboctara equation has numerical solutions that are provided and displayed as 3D plots as shown in Figures 1,2 and 3. So, this transformation is recommended to solve both linear and nonlinear partial differential equations.

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